3.10 iNumbers
A Practice Understanding Task

In order to find solutions to all quadratic equations, we have had to extend the number system to include complex numbers.

Do the following for each of the problems below:
- Choose the best word to complete each conjecture.
- After you have made a conjecture, create at least four examples to show why your conjecture is true.
- If you find a counter-example, change your conjecture to fit your work.

Conjecture #1: The sum of two integers is [always, sometime, never] an integer.

Conjecture #2: The sum of two rational numbers is [always, sometimes, never] a rational number.

Conjecture #3: The sum of two irrational numbers is [always, sometimes, never] an irrational number.
Conjecture #4: The sum of two real numbers is [always, sometimes, never] a real number.

Conjecture #5: The sum of two complex numbers is [always, sometimes, never] a complex number.

Conjecture #6: The product of two integers is [always, sometime, never] an integer.

Conjecture #7: The quotient of two integers is [always, sometime, never] an integer.

Conjecture #8: The product of two rational numbers is [always, sometimes, never] a rational number.

Conjecture #9: The quotient of two rational numbers is [always, sometimes, never] a rational number

Conjecture #10: The product of two irrational numbers is [always, sometimes, never] an irrational number.
Conjecture #11: The product of two real numbers is [always, sometimes, never] a real number.

Conjecture #12: The product of two complex numbers is [always, sometimes, never] a complex number.

13. The ratio of the circumference of a circle to its diameter is given by the irrational number $\pi$. Can the diameter of a circle and the circumference of the same circle both be rational numbers? Explain why or why not.

**The Arithmetic of Polynomials**

In the task *To Be Determined* . . . we defined polynomials to be expressions of the following form:

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-3}x^3 + a_{n-2}x^2 + a_{n-1}x + a_n$$

where all of the exponents are positive integers and all of the coefficients $a_0 \ldots a_n$ are constants.

Do the following for each of the problems below:

- Choose the best word to complete each conjecture.
- After you have made a conjecture, create at least four examples to show why your conjecture is true.
- If you find a counter-example, change your conjecture to fit your work.

Conjecture #P1: The sum of two polynomials is [always, sometime, never] a polynomial.
Conjecture #P2: The difference of two polynomials is [always, sometime, never] a polynomial.

Conjecture #P3: The product of two polynomials is [always, sometime, never] a polynomial.